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THE N=2 VECTOR-TENSOR MULTIPLET, CENTRAL CHARGE SUPERSPACE, AND CHERN-SIMONS COUPLINGS

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Abstract

We present a new, alternative interpretation of the vector-tensor multiplet as a 2-form in central charge superspace. This approach provides a geometric description of the (non-trivial) central charge transformations *ab initio* and is naturally generalized to include couplings of Chern-Simons forms to the antisymmetric tensor gauge field, giving rise to a $N = 2$ supersymmetric version of the Green-Schwarz anomaly cancellation mechanism.

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1 Introduction

As its name already indicates, the vector-tensor multiplet contains among its bosonic components two abelian gauge potentials, a vector and an antisymmetric tensor, subject to intriguing nontrivial central charge transformations: the vector transforms into the dual of the fieldstrength of the antisymmetric tensor and the antisymmetric tensor transforms into the dual fieldstrength of the vector.

These (and other) features have been established in terms of component fields in the original work of Sohnius, Stelle and West [1] and rederived in the context of string compactification [2] more recently.

As to a superspace formulation, the nontrivial structure in the central charge sector suggests a formulation in central charge superspace, *i.e.* a generalization of ordinary superspace with additional bosonic directions corresponding to the central charges, as described by Sohnius [3, 4]. Recall that in this language, ordinary space-time translation as well as supersymmetry and central charge transformations are described as generalized translations in superspace.

The fact that the vector-tensor multiplet contains two gauge potentials of different nature suggests that there are two possibilities to describe it in the framework of superspace geometry.

On the one hand, as has been explained in [5], one may start from an abelian 1-form gauge potential in central charge superspace with judiciously chosen constraints and identify the 2-form gauge potential in this geometric structure *a posteriori*.

On the other hand, one may start from a generic 2-form gauge potential in $N = 2$ central charge superspace, as will be described in the present paper, and identify the 1-form gauge potential as a certain substructure.

As will become clear in the next chapter, this latter formulation provides the veritable generalization of the well-known linear superfield formalism of $N = 1$ supersymmetry to the case of $N = 2$. Moreover, as will be explained in the third chapter, it allows to incorporate central charge transformations *ab origine* at the geometric level. Most importantly, this kind of superspace formulation provides the natural setting for the coupling of Chern-Simons forms to the antisymmetric gauge potential: in chapter **4** we recall the basic properties of Chern-Simons forms together with their couplings to the 2-form geometry in central charge superspace. Finally, in chapter **5** we present the complete description of the coupling of Yang-Mills Chern-Simons forms to the vector-tensor multiplet.

2 2-form gauge potential in central charge superspace

Central charge superspace [4] is a generalization of usual $N = 2$ superspace[1], which, in addition to the vectorial and spinorial coordinates x^a and $\theta^\alpha_{\mathrm{A}}$, $\bar\theta^{\mathrm{A}}_{\dot\alpha}$ contains coordinates z and $\bar z$ referring to the central charge directions. Introducing the notation

$$Z^{\mathcal{A}} = (x^a, \theta^\alpha_{\mathrm{A}}, \bar\theta^{\mathrm{A}}_{\dot\alpha}, z, \bar z), \tag{2.1}$$

one defines the frame (vielbein) $E^{\mathcal{A}}$ in this superspace to have the property

$$dE^{\mathcal{A}} \;=\; T^{\mathcal{A}} \;=\; \frac{1}{2} E^{\mathcal{B}} E^{\mathcal{C}}\, T_{\mathcal{C}\mathcal{B}}{}^{\mathcal{A}}. \tag{2.2}$$

[1] as to the conventional $N = 2$ superspace without central charge our notations follow those of ref. [6]

As usual in superspace geometry, the constant translational torsion is defined as

$$T_{\gamma \mathrm{B}}^{\mathrm{C}\dot{\beta}\,a} \;=\; -2i\,\delta_{\mathrm{B}}^{\mathrm{C}}(\sigma^a\epsilon)_\gamma{}^{\dot{\beta}}, \tag{2.3}$$

whereas the non-vanishing central charge torsion components are

$$T_{\gamma\beta}^{\mathrm{CB}\,z} \;=\; \epsilon_{\gamma\beta}\,\epsilon^{\mathrm{CB}}c^z, \qquad T^{\dot{\gamma}\dot{\beta}\,\bar{z}}_{\mathrm{CB}} \;=\; \epsilon^{\dot{\gamma}\dot{\beta}}\,\epsilon_{\mathrm{CB}}\,\bar{c}^{\bar{z}}, \tag{2.4}$$

with c^z and $\bar{c}^{\bar{z}}$ constant and related to each other by complex conjugation. These constant torsions appear in the anticommutators of the covariant spinorial derivatives, *i.e.*

$$\{D_\gamma^{\mathrm{C}}, D^{\dot{\beta}}_{\mathrm{B}}\} \;=\; 2i\,\delta_{\mathrm{B}}^{\mathrm{C}}(\sigma^a\epsilon)_\gamma{}^{\dot{\beta}}\frac{\partial}{\partial x^a}, \tag{2.5}$$

$$\{D_\gamma^{\mathrm{C}}, D_\beta^{\mathrm{B}}\} \;=\; -\,\epsilon_{\gamma\beta}\,\epsilon^{\mathrm{CB}}\,c^z\frac{\partial}{\partial z}, \qquad \{D^{\dot{\gamma}}_{\mathrm{C}}, D^{\dot{\beta}}_{\mathrm{B}}\} \;=\; -\,\epsilon^{\dot{\gamma}\dot{\beta}}\,\epsilon_{\mathrm{CB}}\,\bar{c}^{\bar{z}}\frac{\partial}{\partial \bar{z}}. \tag{2.6}$$

Given this basic superspace structure we define a 2-form gauge potential B subject to gauge transformations $B' \;=\; B + d\Lambda$ with Λ a superspace 1-form and invariant fieldstrength $H \;=\; dB$. The geometry of the central charge superspace 3-form

$$H \;=\; \frac{1}{3!}\,E^{\mathcal{A}}E^{\mathcal{B}}E^{\mathcal{C}}\,H_{\mathcal{CBA}}, \tag{2.7}$$

is constrained in such a way that all its components are expressed in terms of one single, real, superfield L. This basic covariant superfield is identified in

$$H_{\gamma\mathrm{B}\,a}^{\mathrm{C}\dot{\beta}} \;=\; -2i\,\delta_{\mathrm{B}}^{\mathrm{C}}\;(\sigma_a\epsilon)_\gamma{}^{\dot{\beta}}\;L, \tag{2.8}$$

wheras the other nonvanishing components are given as

$$c^z H_{z\mathrm{BA}}^{\;\dot{\beta}\dot{\alpha}} \;=\; -8\,\epsilon^{\dot{\beta}\dot{\alpha}}\epsilon_{\mathrm{BA}}L, \qquad \bar{c}^{\bar{z}}H_{\bar{z}\beta\alpha}^{\;\mathrm{BA}} \;=\; -8\,\epsilon_{\beta\alpha}\epsilon^{\mathrm{BA}}L, \tag{2.9}$$

$$H_{\gamma\;ba}^{\mathrm{C}} \;=\; 2(\sigma_{ba})_\gamma{}^\varphi D_\varphi^{\mathrm{C}}L, \qquad H^{\dot{\gamma}}_{\mathrm{C}\;ba} \;=\; 2(\bar{\sigma}_{ba})^{\dot{\gamma}}{}_{\dot{\varphi}}\,D^{\dot{\varphi}}_{\mathrm{C}}L, \tag{2.10}$$

$$c^z H_{z\;\mathrm{B}\,a}^{\;\dot{\beta}} \;=\; 4i\,(\bar{\sigma}_a\epsilon)^{\dot{\beta}}{}_\beta\,D^\beta_{\mathrm{B}}L, \qquad \bar{c}^{\bar{z}}H_{\bar{z}\;\beta\,a}^{\;\mathrm{B}} \;=\; 4i\,(\sigma_a\epsilon)_\beta{}^{\dot{\beta}}\,D^{\mathrm{B}}_{\dot{\beta}}L, \tag{2.11}$$

$$c^z\bar{c}^{\bar{z}}H_{\bar{z}\,z\;\mathrm{A}}^{\;\;\dot{\alpha}} \;=\; -8\,D^{\dot{\alpha}}_{\mathrm{A}}L, \qquad \bar{c}^{\bar{z}}c^z H_{z\,\bar{z}\;\alpha}^{\;\;\mathrm{A}} \;=\; -8\,D^{\mathrm{A}}_\alpha L, \tag{2.12}$$

$$c^z H_{z\;ba} \;=\; (\epsilon\sigma_{ba})^{\beta\alpha}D_{\beta\mathrm{A}}D^{\mathrm{A}}_\alpha L, \qquad \bar{c}^{\bar{z}}H_{\bar{z}\;ba} \;=\; (\epsilon\bar{\sigma}_{ba})_{\dot{\beta}\dot{\alpha}}\,D^{\dot{\beta}\mathrm{A}}D^{\dot{\alpha}}_{\mathrm{A}}L, \tag{2.13}$$

As to the purely vectorial component H_{cba} we introduce the dual tensor given as $3!\,h^d \;=\; \varepsilon^{dcba}H_{cba}$, which, in turn, is identified in the covariant superfield expansion of L according to

$$[D^{\mathrm{B}}_\beta, D^{\dot{\alpha}}_{\mathrm{A}}]L \;=\; -2\,\delta^{\mathrm{B}}_{\mathrm{A}}\,h_\beta{}^{\dot{\alpha}}. \tag{2.14}$$

More precisely, it is the trace part in the $N = 2$ indices which determines the dual of H_{cba}, while the traceless part in B and A is a constraint equation for the superfield L. In addition, the superspace Bianchi identities give rise to the constraints

$$\sum_{\mathrm{BA}} D_{\dot{\alpha}\mathrm{B}}D^{\dot{\alpha}}_{\mathrm{A}}L \;=\; 0, \qquad \sum^{\mathrm{BA}} D^{\alpha\mathrm{B}}D^{\mathrm{A}}_\alpha L \;=\; 0. \tag{2.15}$$

The geometric structure exhibited so far clearly indicates that the superfield L is the generalization to $N = 2$ extended supersymmetry of the $N = 1$ linear superfield.

Finally, one identifies the dual of H_{cba} as well in

$$c^z \bar{c}^{\bar{z}} H_{\bar{z}\,z\;a} \;=\; 8i\, h_a. \tag{2.16}$$

This completes our discussion of the components $H_{\mathcal{CBA}}$ of the superspace fieldstrength $H = dB$, which are completely expressed in terms of the single superfield L, subject to three second order spinorial constraints.

In order to discuss the z and $\bar{z}$ dependence of this superfield we define now

$$h \;=\; c^z \partial_z L, \qquad \bar{h} \;=\; \bar{c}^{\bar{z}} \partial_{\bar{z}} L. \tag{2.17}$$

As a consequence of the constraint equations on L itself one finds easily the relations

$$D^{\mathrm{A}}_{\alpha} \bar{h} \;=\; 2i\, \partial_{\alpha}{}^{\dot{\alpha}} D^{\mathrm{A}}_{\dot{\alpha}} L, \qquad D^{\dot{\alpha}}_{\mathrm{A}} h \;=\; 2i\, \partial^{\dot{\alpha}}{}_{\alpha} D^{\alpha}_{\mathrm{A}} L. \tag{2.18}$$

Moreover, applying another spinorial derivative one arrives at

$$c^z \partial_z \bar{h} \;=\; \bar{c}^{\bar{z}} \partial_{\bar{z}} h \;=\; \bar{c}^{\bar{z}} c^z \partial_z \partial_{\bar{z}} L \;=\; 4\, \Box L. \tag{2.19}$$

Supersymmetry as well as central charge transformations will be obtained from covariant translations in the supercoordinates $Z^{\mathcal{A}}$.

$$Z^{\mathcal{A}} \mapsto Z^{\mathcal{A}} + \zeta^{\mathcal{A}}, \tag{2.20}$$

with

$$\zeta^{\mathcal{A}} \;=\; \left(\zeta^a, \zeta^{\alpha}_{\mathrm{A}}, \zeta^{\mathrm{A}}_{\dot{\alpha}}, \zeta^z, \zeta^{\bar{z}}\right) \tag{2.21}$$

When acting on differential forms, we employ the superspace Lie-derivative

$$L_{\zeta} \;=\; \imath_{\zeta}\, d + d\, \imath_{\zeta}, \tag{2.22}$$

with the inner product acting as an antiderivative such that $\imath_{\zeta} E^{\mathcal{A}} = \zeta^{\mathcal{A}}$. As a consequence the 2-form gauge potential changes under the combination of such a diffeomorphism and a 1-form gauge transformation as

$$B \;\mapsto\; B + L_{\zeta} B + d\beta \;=\; B + \imath_{\zeta} H + d(\beta + \imath_{\zeta} B), \tag{2.23}$$

and we define, as usual, covariant superspace translations such that

$$B \;\mapsto\; B + \delta B, \qquad \text{with} \qquad \delta B \;=\; \imath_{\zeta} H. \tag{2.24}$$

This combination of a superspace diffeomorphism and a field dependent compensating 1-form gauge transformation is customarily applied to the description of supersymmetry transformations. When applied to central charge transformations one simply has

$$\delta B \;=\; \imath_{\zeta^z} H + \imath_{\zeta^{\bar{z}}} H. \tag{2.25}$$

Specifying to the components B_{ba} and B_{za}, $B_{\bar{z}a}$ one simply obtains

$$\delta B_{ba} \;=\; \zeta^z H_{zba} + \zeta^{\bar{z}} H_{\bar{z}ba}, \tag{2.26}$$

and

$$\delta B_{za} \;=\; \zeta^{\bar{z}} H_{\bar{z}za}, \qquad \delta B_{\bar{z}a} \;=\; \zeta^z H_{z\bar{z}a}, \tag{2.27}$$

relations which will be useful in a short while in the context of central charge transformations of the vector-tensor multiplet.

3 The vector-tensor multiplet from 2-form geometry

The component field content of the superspace geometry described in the previous section is slightly more general than that of the vector-tensor multiplet. Although the antisymmetric tensor gauge potential is included manifestly, the abelian vector gauge potential remains to be identified. For this purpose we recall that the component fields of the vector tensor multiplet are supposed to depend on the central charge parameters only through the combination $z + \bar{z}$.

We shall implement this particular property here in considering the superspace 1-forms $V = \imath_{c^z} B = E^{\mathcal{A}} c^z B_{z\mathcal{A}}$ and $\bar{V} = \imath_{\bar{c}^{\bar{z}}} B = E^{\mathcal{A}} \bar{c}^{\bar{z}} B_{\bar{z}\mathcal{A}}$ in the particular combination $A = \bar{V} - V$. In view of these identifications we then require $L_{c^z} B = L_{\bar{c}^{\bar{z}}} B$, and obtain, as a consequence,

$$dA \;=\; d(\imath_{\bar{c}^{\bar{z}}} B - \imath_{c^z} B) \;=\; \imath_{c^z} H - \imath_{\bar{c}^{\bar{z}}} H \;=\; F. \tag{3.1}$$

It should be clear that the reality condition $L_{c^z} = L_{\bar{c}^{\bar{z}}}$ applies to any superfield appearing in our geometry, leading, in particular to the identification $\bar{h} = h$ of the superfields defined above.

At the same time this specification provides the missing link which allows now to identify the component fields of the vector-tensor multiplet as well as their supersymmetry and their central charge transformations.

As to the multiplet itself, we identify, in the usual way, component fields as lowest components of superfields, *viz.*

$$\begin{aligned} L| &= L(x), & D^{\text{A}}_{\alpha} L| &= \Lambda^{\text{A}}_{\alpha}(x), & A_a| &= A_a(x), \\ h| &= D(x), & D^{\dot{\alpha}}_{\text{A}} L| &= \bar{\Lambda}^{\dot{\alpha}}_{\text{A}}(x), & B_{ba}| &= B_{ba}(x). \end{aligned}$$

Supersymmetry transformations are determined as

$$\delta L \;=\; \zeta^{\alpha}_{\text{A}} \Lambda^{\text{A}}_{\alpha} + \zeta^{\text{A}}_{\dot{\alpha}} \bar{\Lambda}^{\dot{\alpha}}_{\text{A}}, \tag{3.2}$$

$$\delta \Lambda^{\text{A}}_{\alpha} \;=\; \zeta^{\text{A}}_{\dot{\alpha}} (\bar{\sigma}^a \epsilon)^{\dot{\alpha}}{}_{\alpha}\, i\partial_a L - \tfrac{1}{2}\, (\sigma^{ba})_{\alpha}{}^{\beta} \zeta^{\text{A}}_{\beta}\, \partial_b A_a - \tfrac{1}{2} \zeta^{\text{A}}_{\dot{\alpha}} (\bar{\sigma}_d \epsilon)^{\dot{\alpha}}{}_{\alpha}\, \varepsilon^{dcba} \partial_c B_{ba} - \tfrac{1}{2}\, \zeta^{\text{A}}_{\alpha} D, \tag{3.3}$$

$$\delta \bar{\Lambda}^{\dot{\alpha}}_{\text{A}} \;=\; \zeta^{\alpha}_{\text{A}} (\sigma^a \epsilon)_{\alpha}{}^{\dot{\alpha}}\, i\partial_a L + \tfrac{1}{2}\, (\bar{\sigma}^{ba})^{\dot{\alpha}}{}_{\dot{\beta}} \zeta^{\dot{\beta}}_{\text{A}}\, \partial_b A_a + \tfrac{1}{2}\, \zeta^{\alpha}_{\text{A}} (\sigma_d \epsilon)_{\alpha}{}^{\dot{\alpha}}\, \varepsilon^{dcba} \partial_c B_{ba} - \tfrac{1}{2}\, \zeta^{\dot{\alpha}}_{\text{A}} D, \tag{3.4}$$

$$\delta A_a \;=\; -4i\, \zeta^{\alpha}_{\text{A}} (\sigma_a \epsilon)_{\alpha}{}^{\dot{\alpha}} \Lambda^{\text{A}}_{\dot{\alpha}} + 4i\, \zeta^{\text{A}}_{\dot{\alpha}} (\bar{\sigma}_a \epsilon)^{\dot{\alpha}}{}_{\alpha} \bar{\Lambda}^{\dot{\alpha}}_{\text{A}}, \tag{3.5}$$

$$\delta B_{ba} \;=\; 2\, \zeta^{\beta}_{\text{A}} (\sigma_{ba})_{\beta}{}^{\alpha} \Lambda^{\text{A}}_{\alpha} + 2\, \zeta^{\text{A}}_{\dot{\beta}} (\bar{\sigma}_{ba})^{\dot{\beta}}{}_{\dot{\alpha}} \bar{\Lambda}^{\dot{\alpha}}_{\text{A}}, \tag{3.6}$$

$$\delta D \;=\; -2i\, \zeta^{\alpha}_{\text{A}} (\sigma^a \epsilon)_{\alpha}{}^{\dot{\alpha}}\, \partial_a \bar{\Lambda}^{\text{A}}_{\dot{\alpha}} - 2i\, \zeta^{\text{A}}_{\dot{\alpha}} (\bar{\sigma}^a \epsilon)^{\dot{\alpha}}{}_{\alpha}\, \partial_a \Lambda^{\alpha}_{\text{A}}. \tag{3.7}$$

Since central charge transformations are understood as translations in z, $\bar{z}$ space we shall adapt them to the central charge reality condition in employing the parametrization $\zeta^z = wc^z$ and $\zeta^{\bar{z}} = w\bar{c}^{\bar{z}}$. Covariant central charge translations of the gauge potentials A and B are then defined as combinations of these constant diffeomorphisms and, again, properly identified compensating field dependent gauge transformations such that

$$\delta_{c.c.} A \;=\; \imath_{\zeta^z} F + \imath_{\zeta^{\bar{z}}} F \;=\; -2w\, \imath_{c^z} \imath_{\bar{c}^{\bar{z}}} H, \tag{3.8}$$

$$\delta_{c.c.} B \;=\; \imath_{\zeta^z} H + \imath_{\zeta^{\bar{z}}} H \;=\; w\, (\imath_{c^z} H + \imath_{\bar{c}^{\bar{z}}} H). \tag{3.9}$$

Projecting to the vectorial components of these superspace forms and using previous results and identifications one finds that the vector gauge potential transforms into the dual of the

fieldstrength of the tensor potential, whereas the tensor potential transforms into the dual of the fieldstrength of the vector potential, in more explicit terms

$$\delta_{c.c.} A_a \;=\; -8i\, w\, \epsilon_{abcd}\, \partial^b B^{cd}, \tag{3.10}$$

$$\delta_{c.c.} B_{ab} \;=\; i\, w\, \epsilon_{abcd}\, \partial^c A^d. \tag{3.11}$$

As the remaining component fields were identified as lowest components of covariant superfields their central charge transformations are obtained straightforwardly to be

$$\delta_{c.c.} L \;=\; 2\, w\, D, \tag{3.12}$$

$$\delta_{c.c.} \Lambda_\alpha^{\mathrm{A}} \;=\; -4i\, w\, (\sigma^a \epsilon)_\alpha{}^{\dot\alpha}\, \partial_a \bar\Lambda^{\mathrm{A}}_{\dot\alpha}, \tag{3.13}$$

$$\delta_{c.c.} \bar\Lambda^{\dot\alpha}_{\mathrm{A}} \;=\; -4i\, w\, (\bar\sigma^a \epsilon)^{\dot\alpha}{}_\alpha\, \partial_a \Lambda^\alpha_{\mathrm{A}}, \tag{3.14}$$

$$\delta_{c.c.} D \;=\; 8\, w\, \Box L. \tag{3.15}$$

This concludes our discussion of the vector-tensor multiplet in the new context of 2-form central charge geometry. The invariant superfield and component field actions will be presented below after having included coupling to Chern-Simons forms.

4 2-form geometry and Chern-Simons forms

In order to prepare the ground for the coupling of Chern-Simons forms we return now to the more general setting of section **2**, *i.e.* without reality conditions as to the z, $\bar z$ dependence. We consider a Yang-Mills theory gauge potential $\mathcal{A} \;=\; E^{\mathcal{A}} \mathcal{A}_{\mathcal{A}}$ together with its fieldstrength

$$\mathcal{F} \;=\; d\mathcal{A} + \mathcal{A}\mathcal{A} \;=\; \frac{1}{2} E^{\mathcal{B}} E^{\mathcal{C}}\, \mathcal{F}_{\mathcal{C}\mathcal{B}} \;=\; E^{\mathcal{B}} E^{\mathcal{C}} \left(D_{\mathcal{C}} \mathcal{A}_{\mathcal{B}} - \mathcal{A}_{\mathcal{C}} \mathcal{A}_{\mathcal{B}} + \frac{1}{2} T_{\mathcal{C}\mathcal{B}}{}^{\mathcal{A}} \mathcal{A}_{\mathcal{A}} \right), \tag{4.1}$$

satisfying Bianchi identities $\mathcal{D}\mathcal{F} \;=\; 0$, in some more detail

$$E^{\mathcal{A}} E^{\mathcal{B}} E^{\mathcal{C}} \left(\mathcal{D}_{\mathcal{C}} \mathcal{F}_{\mathcal{B}\mathcal{A}} + T_{\mathcal{C}\mathcal{B}}{}^{\mathcal{F}} \mathcal{F}_{\mathcal{F}\mathcal{A}} \right) \;=\; 0. \tag{4.2}$$

As is well-known, the covariant constraints

$$\mathcal{F}^{\mathrm{BA}}_{\beta\alpha} \;=\; 0, \qquad \mathcal{F}^{\dot\beta\dot\alpha}_{\mathrm{BA}} \;=\; 0, \qquad \mathcal{F}^{\mathrm{B}\dot\alpha}_{\beta\mathrm{A}} \;=\; 0, \tag{4.3}$$

allow to express the components of $\mathcal{F}$ entirely in terms of the basic spinorial gaugino superfields $\mathcal{W}^{\mathrm{A}}_\alpha$ and $\bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}}$. They are identified in

$$\mathcal{F}^{\mathrm{B}}_{\beta\, a} \;=\; +i(\sigma_a \epsilon)_\beta{}^{\dot\beta}\, \bar{\mathcal{W}}^{\mathrm{B}}_{\dot\beta}, \qquad \mathcal{F}^{\dot\beta}_{\mathrm{B}\, a} \;=\; -i(\bar\sigma_a \epsilon)^{\dot\beta}{}_\beta\, \mathcal{W}^\beta_{\mathrm{B}}, \tag{4.4}$$

and appear as well in

$$c^z \mathcal{F}_{z\mathrm{B}}{}^{\dot\beta} \;=\; -4\, \bar{\mathcal{W}}^{\dot\beta}_{\mathrm{B}}, \qquad \bar c^{\bar z} \mathcal{F}_{\bar z\beta}{}^{\mathrm{B}} \;=\; +4\, \mathcal{W}^{\mathrm{B}}_\beta. \tag{4.5}$$

The other nonvanishing components of the fieldstrength, as well as the derivative constraints on $\mathcal{W}^{\mathrm{A}}_\alpha$ and $\bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}}$, are summarized in

$$\mathcal{D}^{\mathrm{B}}_\beta\, \mathcal{W}^{\mathrm{A}}_\alpha \;=\; +\epsilon^{\mathrm{BA}} (\sigma^{ba}\epsilon)_{\beta\alpha}\, \mathcal{F}_{ba} + \epsilon_{\beta\alpha} \sum^{\mathrm{BA}} \mathcal{D}^{\mathrm{B}}_{\dot\varphi} \bar{\mathcal{W}}^{\dot\varphi\mathrm{A}} - \frac{1}{8} \epsilon_{\beta\alpha} \epsilon^{\mathrm{BA}} c^z \bar c^{\bar z} \mathcal{F}_{\bar z z}, \tag{4.6}$$

$$\mathcal{D}^{\dot\beta}_{\mathrm{B}}\, \bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}} \;=\; -\epsilon_{\mathrm{BA}} (\bar\sigma^{ba}\epsilon)^{\dot\beta\dot\alpha}\, \mathcal{F}_{ba} + \epsilon^{\dot\beta\dot\alpha} \sum_{\mathrm{BA}} \mathcal{D}^\varphi_{\mathrm{B}} \mathcal{W}_{\varphi\mathrm{A}} + \frac{1}{8} \epsilon^{\dot\beta\dot\alpha} \epsilon_{\mathrm{BA}} \bar c^{\bar z} c^z \mathcal{F}_{z\bar z}. \tag{4.7}$$

Moreover, one has

$$\mathcal{D}^{\mathrm{B}}_{\beta}\,\bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}} \;=\; -\frac{i}{2}\delta^{\mathrm{B}}_{\mathrm{A}}\,(\sigma^a\epsilon)_{\beta}{}^{\dot\alpha}\; c^z\mathcal{F}_{z\,a}, \qquad \mathcal{D}^{\dot\beta}_{\mathrm{B}}\,\mathcal{W}^{\mathrm{A}}_{\alpha} \;=\; +\frac{i}{2}\delta^{\mathrm{A}}_{\mathrm{B}}\,(\bar\sigma^a\epsilon)^{\dot\beta}{}_{\alpha}\;\bar c^{\bar z}\mathcal{F}_{\bar z\,a}. \tag{4.8}$$

Having established the general scenario for the Yang-Mills gauge structure we shall now define the corresponding Chern-Simons forms as

$$\mathcal{Q} \;=\; \mathrm{tr}(\mathcal{A}\mathcal{F} - \tfrac{1}{3}\mathcal{A}\mathcal{A}\mathcal{A}), \tag{4.9}$$

which are coupled to the 2-form geometry as described in section **2** according to

$$\mathcal{H} \;=\; dB + k\mathcal{Q}. \tag{4.10}$$

The modified Bianchi-identities $d\mathcal{H} = k\,\mathrm{tr}(\mathcal{F}\mathcal{F})$ for this case result in a number of modifications in the components of $H_{\mathcal{CBA}}$ and the constraints on L compared to the results in section **2**. One finds in particular

$$c^z\mathcal{H}_{z\;ba} \;=\; (\epsilon\sigma_{ba})^{\beta\alpha}D_{\beta\mathrm{A}}D^{\mathrm{A}}_{\alpha}L - 2k\,(\epsilon\bar\sigma_{ba})_{\dot\beta\dot\alpha}\,\mathrm{tr}\left(\bar{\mathcal{W}}^{\dot\beta\mathrm{A}}\bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}}\right), \tag{4.11}$$

$$\bar c^{\bar z}\mathcal{H}_{\bar z\;ba} \;=\; (\epsilon\bar\sigma_{ba})_{\dot\beta\dot\alpha}\,D^{\dot\beta\mathrm{A}}D^{\dot\alpha}_{\mathrm{A}}L - 2k\,(\epsilon\sigma_{ba})^{\beta\alpha}\,\mathrm{tr}\left(\mathcal{W}_{\beta\mathrm{A}}\mathcal{W}^{\mathrm{A}}_{\alpha}\right), \tag{4.12}$$

as well as

$$[D^{\mathrm{B}}_{\beta}, D^{\dot\alpha}_{\mathrm{A}}]L \;=\; -2\,\delta^{\mathrm{B}}_{\mathrm{A}}\,\hat h_{\beta}{}^{\dot\alpha} - 4k\,\mathrm{tr}\left(\mathcal{W}^{\mathrm{B}}_{\beta}\bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}}\right), \tag{4.13}$$

with

$$\hat h_{\beta}{}^{\dot\alpha} \;=\; h_{\beta}{}^{\dot\alpha} - 2k\,\mathrm{tr}\left(\mathcal{W}^{\mathrm{A}}_{\beta}\bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}}\right). \tag{4.14}$$

Furthermore

$$\sum_{\mathrm{BA}} D_{\dot\alpha\mathrm{B}}D^{\dot\alpha}_{\mathrm{A}}L \;=\; 4k\,\mathrm{tr}\left(\mathcal{W}^{\alpha}_{\mathrm{B}}\mathcal{W}_{\alpha\mathrm{A}}\right), \tag{4.15}$$

$$\sum^{\mathrm{BA}} D^{\alpha\mathrm{B}}D^{\mathrm{A}}_{\alpha}L \;=\; 4k\,\mathrm{tr}\left(\bar{\mathcal{W}}^{\mathrm{B}}_{\dot\alpha}\bar{\mathcal{W}}^{\dot\alpha\mathrm{A}}\right). \tag{4.16}$$

Finally,

$$c^z\bar c^{\bar z}H_{\bar z\,z\;a} \;=\; 8i\,\hat h_a, \tag{4.17}$$

while the components $H^{\mathrm{A}}_{\alpha\,z\bar z}$ and $H^{\dot\alpha}_{\mathrm{A}\;z\bar z}$ are given as before.

5 Vector-tensor multiplet and coupling to Chern-Simons forms

The Yang-Mills multiplet of ref.[7] is recovered from the more general geometric structure described in the previous section in requiring all superfields (in the Yang-Mills sector) to be independent of z and $\bar z$. This means in particular that the Yang-Mills gauge potentials having vanishing Lie derivatives in the directions of c^z and $\bar c^{\bar z}$,

$$L_{c^z}\mathcal{A} \;=\; L_{\bar c^{\bar z}}\mathcal{A} \;=\; 0, \tag{5.1}$$

and similarly for the gauge parameter superfields. As a consequence, the components $\mathcal{A}_z$ and $\mathcal{A}_{\bar z}$ become gauge covariant superfields. Written in covariant form the same conditions become

$$\imath_{c^z}\mathcal{F} \;=\; -D\bar X, \qquad \imath_{\bar c^{\bar z}}\mathcal{F} \;=\; -DX, \tag{5.2}$$

where we have defined $c^z A_z = \bar{X}$ and $\bar{c}^{\bar{z}} A_{\bar{z}} = X$. More explicitly, one obtains the familiar relations[2]

$$\mathcal{D}^{\mathrm{A}}_{\alpha} \bar{X} \; = \; 0, \qquad \mathcal{D}^{\mathrm{A}}_{\alpha} X \; = \; -4\, \mathcal{W}^{\mathrm{A}}_{\alpha}, \tag{5.3}$$

$$\mathcal{D}^{\dot{\alpha}}_{\mathrm{A}} \bar{X} \; = \; +4\, \bar{\mathcal{W}}^{\dot{\alpha}}_{\mathrm{A}}, \qquad \mathcal{D}^{\dot{\alpha}}_{\mathrm{A}} X \; = \; 0. \tag{5.4}$$

Having specified the Yang-Mills gauge structure, whose Chern-Simons form is to be coupled to the 2-form of the vector-tensor multiplet, we have now to reconsider the identification of the abelian vector gauge potential in the presence of Chern-Simons forms.

In order to get some intuition we shall consider first infinitesimal Yang-Mills gauge transformations which change the gauge potential $\mathcal{A}$ and its Chern-Simons form $\mathcal{Q}$ according to

$$\delta\mathcal{A} \; = \; -d\alpha - [\alpha, \mathcal{A}], \qquad \text{and} \qquad \delta\mathcal{Q} \; = \; d\,\mathrm{tr}(\mathcal{A}\, d\alpha). \tag{5.5}$$

The variation of the Chern-Simons form in the fieldstrength $\mathcal{H} = dB + k\mathcal{Q}$ can be compensated by assigning the transformation law

$$\delta B \; = \; d\beta - k\,\mathrm{tr}(\mathcal{A}\, d\alpha) \tag{5.6}$$

to the 2-form gauge potential. Given this modified transformation law we define the 1-forms

$$\mathcal{V} \; = \; \imath_{c^z} B + k\,\mathrm{tr}(\mathcal{A}\, \imath_{c^z} \mathcal{A}), \qquad \bar{\mathcal{V}} \; = \; \imath_{\bar{c}^{\bar{z}}} B + k\,\mathrm{tr}(\mathcal{A}\, \imath_{\bar{c}^{\bar{z}}} \mathcal{A}), \tag{5.7}$$

subject to gauge transformations

$$\delta\mathcal{V} \; = \; L_{c^z}\beta - d\,\imath_{c^z}\beta, \qquad \delta\bar{\mathcal{V}} \; = \; L_{\bar{c}^{\bar{z}}}\beta - d\,\imath_{\bar{c}^{\bar{z}}}\beta. \tag{5.8}$$

Requiring $L_{c^z}\beta = L_{\bar{c}^{\bar{z}}}\beta$ as well as $L_{c^z}B = L_{\bar{c}^{\bar{z}}}B$ we define then the abelian 1-form gauge potential $A = \bar{\mathcal{V}} - \mathcal{V}$, whose fieldstrength

$$F \; = \; dA \; = \; \imath_{c^z}\mathcal{H} - \imath_{\bar{c}^{\bar{z}}}\mathcal{H} - 2k\,\mathrm{tr}((\bar{X} - X)\mathcal{F}), \tag{5.9}$$

is invariant under the gauge transformations $\delta A = d(\imath_{c^z}\beta - \imath_{\bar{c}^{\bar{z}}}\beta)$. As a consequence of their definition, the fieldstrength components $F_{\mathcal{BA}}$ are completely expressed in terms of the superfields L and X, $\bar{X}$. In particular, the non-vanishing components at canonical dimensions 0 and 1/2 are given as

$$F^{\mathrm{BA}}_{\beta\alpha} \; = \; +8\epsilon_{\beta\alpha}\epsilon^{\mathrm{BA}} L, \qquad F^{\dot{\beta}\dot{\alpha}}_{\mathrm{BA}} \; = \; -8\epsilon^{\dot{\beta}\dot{\alpha}}\epsilon_{\mathrm{BA}} L, \tag{5.10}$$

$$F^{\mathrm{B}}_{\beta\, a} \; = \; +i(\sigma_a\epsilon)_{\beta}{}^{\dot{\beta}}\, \bar{\Gamma}^{\mathrm{B}}_{\dot{\beta}}, \qquad F^{\dot{\beta}}_{\mathrm{B}\, a} \; = \; -i(\bar{\sigma}_a\epsilon)^{\dot{\beta}}{}_{\beta}\, \Gamma^{\beta}_{\mathrm{B}}, \tag{5.11}$$

$$c^z F_z{}^{\mathrm{A}}_{\alpha} \; = \; -\bar{c}^{\bar{z}} F_{\bar{z}}{}^{\mathrm{A}}_{\alpha} + 4\Gamma^{\mathrm{A}}_{\alpha} \; = \; -8D^{\mathrm{A}}_{\alpha} L, \qquad \bar{c}^{\bar{z}} F_{\bar{z}}{}^{\dot{\alpha}}_{\mathrm{A}} \; = \; -c^z F_z{}^{\dot{\alpha}}_{\mathrm{A}} - 4\bar{\Gamma}^{\dot{\alpha}}_{\mathrm{A}} \; = \; +8D^{\dot{\alpha}}_{\mathrm{A}} L. \tag{5.12}$$

Here, we have defined the superfield

$$\Gamma = L + \frac{k}{16}\mathrm{tr}(\bar{X} - X)^2, \tag{5.13}$$

[2]The superspace geometry of [7] is then easily obtained from an appropriate adjustement of conventional constraints as explained in [3, 4].

together with its spinorial derivatives $\Gamma^{\beta}_{\mathrm{B}} = -4D^{\beta}_{\mathrm{B}}\Gamma$ and $\bar{\Gamma}^{\mathrm{B}}_{\dot{\beta}} = -4D^{\mathrm{B}}_{\dot{\beta}}\Gamma$. Moreover, due this definition and the special properties of the superfields L and X, $\bar{X}$ induced from the 2-form and Yang-Mills geometries, respectively, one obtains

$$\sum^{\mathrm{BA}} D^{\alpha\mathrm{B}} D^{\mathrm{A}}_{\alpha}\Gamma \;=\; \sum^{\mathrm{BA}} D^{\mathrm{B}}_{\dot{\alpha}} D^{\dot{\alpha}\mathrm{A}}\Gamma, \tag{5.14}$$

as well as

$$\sum^{\mathrm{BA}} D^{\mathrm{B}}_{\dot{\beta}} D^{\mathrm{A}}_{\alpha}\Gamma \;=\; 0, \qquad \sum^{\mathrm{BA}} D^{\mathrm{A}}_{\alpha} D^{\mathrm{B}}_{\dot{\beta}}\Gamma \;=\; 0. \tag{5.15}$$

Obviously, the remaining components of F, at canonical dimension 1, are related to the superfield expansion of the basic superfields L and X, $\bar{X}$ as well, in particular

$$F_{ba} \;=\; -\frac{1}{4}(\sigma_{ba})_{\alpha}{}^{\beta} D^{\mathrm{B}}_{\beta}\Gamma^{\alpha}_{\mathrm{B}} + \frac{1}{4}(\bar{\sigma}_{ba})^{\dot{\alpha}}{}_{\dot{\beta}} D^{\dot{\beta}}_{\mathrm{B}}\bar{\Gamma}^{\mathrm{B}}_{\dot{\alpha}}. \tag{5.16}$$

With the complete geometric description of Chern-Simons couplings at hand it is now straightforward to write down an invariant action and to identify component fields together with their supersymmetry and central charge transformations.

As to the construction of a supersymmetric action we define the superfield

$$\Sigma \;=\; L - \frac{k}{16}\mathrm{tr}(X+\bar{X})^2. \tag{5.17}$$

It is easy to convince oneself that Σ, similarly to Γ, has the properties

$$\sum^{\mathrm{BA}} D^{\alpha\mathrm{B}} D^{\mathrm{A}}_{\alpha}\Sigma + \sum^{\mathrm{BA}} D^{\mathrm{B}}_{\dot{\alpha}} D^{\dot{\alpha}\mathrm{A}}\Sigma \;=\; 0, \tag{5.18}$$

as well as

$$\sum^{\mathrm{BA}} D^{\mathrm{B}}_{\dot{\beta}} D^{\mathrm{A}}_{\alpha}\Sigma \;=\; 0, \qquad \sum^{\mathrm{BA}} D^{\mathrm{A}}_{\alpha} D^{\mathrm{B}}_{\dot{\beta}}\Sigma \;=\; 0. \tag{5.19}$$

As a consequence, the combination

$$M^{\overset{\mathrm{BA}}{\smile}} \;=\; \sum^{\mathrm{BA}} \Big(D^{\alpha\mathrm{B}}\Sigma\; D^{\mathrm{A}}_{\alpha}\Sigma + \Sigma D^{\alpha\mathrm{B}} D^{\mathrm{A}}_{\alpha}\Sigma - D^{\mathrm{B}}_{\dot{\alpha}}\Sigma\; D^{\dot{\alpha}\mathrm{A}}\Sigma \Big), \tag{5.20}$$

satisfies

$$\oint_{\mathrm{CBA}} D_{\underline{\gamma}\mathrm{C}} M_{\underset{\smile}{\mathrm{BA}}} \;=\; 0, \tag{5.21}$$

for $\underline{\gamma} = \gamma, \dot{\gamma}$, and can, therefore, be employed for the construction of a supersymmetric component field action [5],[10], obtained as the lowest component of the superfield

$$\Big(D^{\alpha\mathrm{B}} D^{\mathrm{A}}_{\alpha} - D^{\mathrm{B}}_{\dot{\alpha}} D^{\dot{\alpha}\mathrm{A}} \Big) M_{\underset{\smile}{\mathrm{BA}}}. \tag{5.22}$$

A straightforward calculation leads then to the component field action (canonical normalization of kinetic terms is easily established in performing suitable field redefinitions)

$$\mathcal{L}_{VT} \;=\; -\frac{1}{2}\,\partial^m L\,\partial_m L + \frac{1}{2}\,\hat{h}^m \hat{h}_m - \frac{1}{64}\,\Sigma^{mn}\Sigma_{mn} + \frac{i}{2}\,(\sigma^m\epsilon)_{\alpha}{}^{\dot{\alpha}}\,\Lambda^{\alpha}_{\mathrm{A}}\,\overleftrightarrow{\partial}_m\,\bar{\Lambda}^{\mathrm{A}}_{\dot{\alpha}} - \frac{1}{8}\,D^2$$

$$+g_{(\mathrm{i})(\mathrm{j})}\Big\{ -\mathcal{D}_m X^{(\mathrm{i})}\,\mathcal{D}^m \bar{X}^{(\mathrm{j})} + 2\,\mathcal{F}^{(\mathrm{i})}_{mn}\,\mathcal{F}^{(\mathrm{j})\,mn} - 4i\,(\sigma^m\epsilon)_{\alpha}{}^{\dot{\alpha}}\,\mathcal{W}^{(\mathrm{i})\alpha}{}_{\mathrm{A}}\,\overleftrightarrow{\mathcal{D}}_m\,\bar{\mathcal{W}}^{(\mathrm{j})\mathrm{A}}{}_{\dot{\alpha}}$$

$$
-\frac{1}{2}\,\mathbf{D}^{(\mathrm{i})\,\underset{\smile}{\mathrm{BA}}}\,\mathbf{D}^{(\mathrm{j})}_{\underset{\smile}{\mathrm{BA}}} - \frac{1}{16}\,[X,\bar{X}]^{(\mathrm{i})}\,[X,\bar{X}]^{(\mathrm{j})} + 2\,\mathcal{W}^{(\mathrm{i})\alpha}{}_{\mathrm{A}}\,[\bar{X},\mathcal{W}^{\mathrm{A}}_{\alpha}]^{(\mathrm{j})} - 2\,\bar{\mathcal{W}}^{(\mathrm{i})\mathrm{A}}{}_{\dot\alpha}\,[X,\bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}}]^{(\mathrm{j})}\Big\}
$$

$$
+\frac{k^2}{2}\,\mathrm{tr}\left((X+\bar{X})\mathcal{W}^{\alpha}_{\mathrm{A}}\right)\sigma^{mn}{}_{\alpha}{}^{\beta}\,\mathrm{tr}\left(\mathcal{W}^{\mathrm{A}}_{\beta}\mathcal{F}_{mn}\right) + \frac{k^2}{2}\,\mathrm{tr}\left((X+\bar{X})\bar{\mathcal{W}}^{\mathrm{A}}_{\dot\alpha}\right)\bar{\sigma}^{mn\dot\alpha}{}_{\dot\beta}\,\mathrm{tr}\left(\bar{\mathcal{W}}^{\dot\beta}_{\mathrm{A}}\mathcal{F}_{mn}\right)
$$

$$
+\frac{k^2}{8}\,\mathcal{W}^{(\mathrm{i})\alpha\mathrm{A}}\sigma^{m}_{\alpha\dot\alpha}\bar{\mathcal{W}}^{(\mathrm{j})\dot\alpha}{}_{\mathrm{A}}\,\mathbf{t}_m^{\underset{\smile}{(\mathrm{i})(\mathrm{j})}} + \frac{1}{2}\,g_{(\mathrm{i})(\mathrm{j})}\,Q^{(\mathrm{i})\,\underset{\smile}{\mathrm{BA}}}\,Q^{(\mathrm{j})}_{\underset{\smile}{\mathrm{BA}}} + \frac{k^2}{4}q^{\underset{\smile}{\mathrm{BA}}}q_{\underset{\smile}{\mathrm{BA}}}.
$$

$$
+\frac{ik}{8}\,\hat{h}^m\,\mathrm{tr}\left((X+\bar{X})\mathcal{D}_m(X-\bar{X})\right) + \frac{ik}{32}\,\varepsilon^{klmn}\Sigma_{kl}\mathrm{tr}\left((X+\bar{X})\mathcal{F}_{mn}\right)
$$

$$
+k\Lambda^{\alpha}_{\mathrm{A}}\,\mathrm{tr}\Big(\sigma^{mn}{}_{\alpha}{}^{\beta}\,\mathcal{W}^{\mathrm{A}}_{\beta}\mathcal{F}_{mn} - \frac{1}{8}[X,\bar{X}]\,\mathcal{W}^{\mathrm{A}}_{\alpha} - \frac{i}{2}\bar{\mathcal{W}}^{\dot\alpha\mathrm{A}}\mathcal{D}_{\alpha\dot\alpha}X\Big)
$$

$$
-k\bar{\Lambda}^{\mathrm{A}}_{\dot\alpha}\,\mathrm{tr}\Big(\bar{\sigma}^{mn\dot\alpha}{}_{\dot\beta}\,\bar{\mathcal{W}}^{\dot\beta}_{\mathrm{A}}\mathcal{F}_{mn} - \frac{1}{8}[X,\bar{X}]\,\bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}} - \frac{i}{2}\bar{\mathcal{W}}_{\alpha\mathrm{A}}\mathcal{D}^{\dot\alpha\alpha}\bar{X}\Big) \tag{5.23}
$$

We have used the convention $\mathrm{tr}(AB) = A^{(\mathrm{i})}B^{(\mathrm{i})}$ and the abbreviations

$$
\mathbf{t}_m^{\underset{\smile}{(\mathrm{i})(\mathrm{j})}} \;=\; i\sum^{(\mathrm{i})(\mathrm{j})}(X+\bar{X})^{(\mathrm{i})}\,\mathcal{D}_m(X-\bar{X})^{(\mathrm{j})}, \tag{5.24}
$$

as well as

$$
q_{\underset{\smile}{\mathrm{BA}}} \;=\; \mathrm{tr}\Big(\bar{\mathcal{W}}_{\dot\alpha\mathrm{B}}\bar{\mathcal{W}}^{\dot\alpha}_{\mathrm{A}} - \mathcal{W}^{\alpha}_{\mathrm{B}}\mathcal{W}_{\alpha\mathrm{A}}\Big), \tag{5.25}
$$

and

$$
Q^{(\mathrm{i})}_{\underset{\smile}{\mathrm{BA}}} \;=\; \bar{g}^{(\mathrm{i})(\mathrm{j})}\,\left(\frac{k^2}{8}(X+\bar{X})^{(\mathrm{j})}q_{\underset{\smile}{\mathrm{BA}}} - \frac{k}{4}\sum_{\mathrm{BA}}\left(\Sigma^{\alpha}_{\mathrm{B}}\,\mathcal{W}^{(\mathrm{j})}_{\alpha\mathrm{A}} + \bar{\Sigma}_{\dot\alpha\mathrm{B}}\bar{\mathcal{W}}^{(\mathrm{j})\dot\alpha}{}_{\mathrm{A}}\right)\right), \tag{5.26}
$$

with $\bar{g}^{(\mathrm{i})(\mathrm{j})}$ the inverse of the gauge coupling function

$$
g_{(\mathrm{i})(\mathrm{j})} \;=\; \frac{k}{4}\left(\Sigma\,\delta_{(\mathrm{i})(\mathrm{j})} - \frac{k}{8}\,(X+\bar{X})_{(\mathrm{i})}(X+\bar{X})_{(\mathrm{j})}\right) \tag{5.27}
$$

and

$$
\Sigma^{\mathrm{A}}_{\alpha} \;=\; D^{\mathrm{A}}_{\alpha}\Sigma, \qquad \bar{\Sigma}^{\dot\alpha}_{\mathrm{A}} \;=\; D^{\dot\alpha}_{\mathrm{A}}\Sigma. \tag{5.28}
$$

Of course, for $k = 0$, one obtains simply the usual free theory for the pure vector-tensor multiplet, which describes a non-interacting theory. Interestingly enough, the coupling to Chern-Simons forms already in itself induces a kinetic term for the Yang-Mills sector, with field-dependent gauge coupling function $g_{(\mathrm{i})(\mathrm{j})}$. Other k-dependent modifications are encoded in the definitions

$$
\hat{h}^k \;=\; \varepsilon^{klmn}\left(\tfrac{1}{2}\partial_l B_{mn} + k\;\mathrm{tr}(\mathcal{A}_l\partial_m\mathcal{A}_n - \tfrac{2}{3}\mathcal{A}_l\mathcal{A}_m\mathcal{A}_n)\right) + k\,\bar{\sigma}^{k\,\dot\alpha\alpha}\,\mathrm{tr}(\mathcal{W}^{\mathrm{A}}_{\alpha}\bar{\mathcal{W}}_{\dot\alpha\mathrm{A}}) \tag{5.29}
$$

and

$$
\Sigma_{mn} \;=\; F_{mn} - 2k\;\mathrm{tr}\Big((X-\bar{X})\mathcal{F}_{mn} - 2\,\mathcal{W}^{\alpha}_{\mathrm{A}}(\sigma_{mn})_{\alpha}{}^{\beta}\mathcal{W}^{\mathrm{A}}_{\beta} + 2\,\bar{\bar{\mathcal{W}}}^{\mathrm{A}}_{\dot\alpha}(\bar{\sigma}_{mn})^{\dot\alpha}{}_{\dot\beta}\bar{\bar{\mathcal{W}}}^{\dot\beta}_{\mathrm{A}}\Big), \tag{5.30}
$$

with the field tensor $F_{mn} \;=\; \partial_m A_n - \partial_n A_m$ identified in the geometric description given above. As to the definitions of the component fields X, $\bar{X}$ and $\mathcal{W}^{\alpha}_{\mathrm{A}}$, $\bar{\mathcal{W}}^{\mathrm{A}}_{\dot\alpha}$ in the Yang-Mills sector we

have deliberately used the same symbols for component fields, identified as lowest component of the corresponding superfields. Moreover, we have

$$\mathcal{F}_{mn} \;=\; \partial_m \mathcal{A}_n - \partial_n \mathcal{A}_m + [\mathcal{A}_m, \mathcal{A}_n], \tag{5.31}$$

and

$$\mathbf{D}^{(\mathrm{i})}_{\underbrace{\scriptstyle\mathrm{BA}}} \;=\; \frac{1}{2} \sum_{\mathrm{BA}} \mathcal{D}^{\alpha}_{\mathrm{B}} \mathcal{D}_{\alpha\mathrm{A}} X^{(\mathrm{i})} + Q^{(\mathrm{i})}_{\underbrace{\scriptstyle\mathrm{BA}}}. \tag{5.32}$$

Supersymmetry and central charge transformations of the component fields which leave the above lagrangian invariant are easily deduced from the underlying geometric framework in superspace and will not be rephrased here explicitly.

6 Conclusion and outlook

We have shown that the vector-tensor multiplet can be obtained from the two-form geometry in central charge superspace with suitably chosen constraints. This formulation allows to derive central charge transformations of the component fields a priori on purely geometric grounds. Moreover, from this point of view, the analogies of the vector-tensor multiplet with the linear multiplet of $N = 1$ supersymmetry become quite evident. This analogy can be pursued in order to construct an interacting theory due to Chern-Simons coupling. Finally, we have strong reasons to believe that this formulation should be quite helpful in the coupling of the vector-tensor multiplet to supergravity and the study of nontrivial central charges in the framework of local supersymmetry.

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